

Honors Physics Test - Ch. 8 - Torque and Equilibrium - 04-28-10a

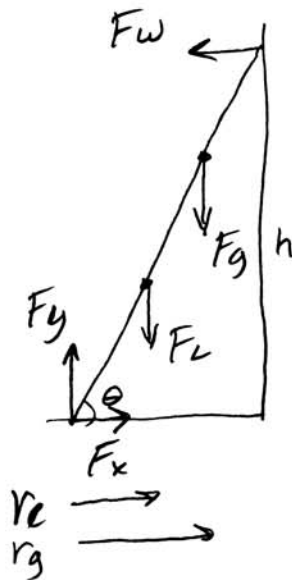
Name WARD _____^o

$\tau = rF$ (if r or a component of r is perpendicular to τ) $x_{cg} = (m_1x_1 + m_2x_2 + \dots) / (m_1 + m_2 + \dots)$
 Equilibrium $\rightarrow \Sigma F = 0 \quad \Sigma \tau = 0 \quad \mu = F_f / F_N$

List your givens and unknown(s) for each problem.

1. A 12 m long, 270 N ladder (whose center of mass is 5 meters up along the length of the ladder) leans against a wall and makes a 75° angle with the ground. A 900 N guy is standing 9 m up along the length of the ladder. (a) Draw a large good diagram showing all forces and r components, carefully labeled. (b) What is the horizontal force of the ground at the foot of the ladder? (c) What is the vertical force at the foot of the ladder? (d) What coefficient of static is needed to prevent the ladder from slipping?

- $l = 12 \text{ m}$
 $F_L = 270 \text{ N}$
 $l_c = 5 \text{ m}$
 $\theta = 75^\circ$
 $F_g = 900 \text{ N}$
 $l_g = 9 \text{ m}$



$$\Sigma \vec{\tau} = 0$$

$$-l_c(\cos\theta)F_c - l_g(\cos\theta)F_g + l(\sin\theta)F_w = 0$$

$$F_w = \frac{l_c(\cos\theta)F_c + l_g(\cos\theta)F_g}{l \sin\theta}$$

$$= \frac{5 \text{ m}(\cos 75^\circ)270 \text{ N} + 9 \text{ m}(\cos 75^\circ)900 \text{ N}}{12 \text{ m}(\sin 75^\circ)}$$

$$F_w = 211 \text{ N}$$

$$\Sigma F_x = 0$$

$$F_x - F_w = 0$$

$$F_x = F_w$$

b) $F_x = 211 \text{ N}$

$$\Sigma \vec{F}_y = 0$$

$$F_y - F_c - F_g = 0$$

$$F_y = F_c + F_g$$

$$= 270 \text{ N} + 900 \text{ N}$$

c) $F_y = 1170 \text{ N}$

$$f_s = \mu_s F_N$$

$$\mu_s = \frac{f_s}{F_N}$$

$$= \frac{211 \text{ N}}{1170 \text{ N}}$$

$$f_s = F_x$$

$$F_N = F_y$$

d) $\mu_s = 0.18$

2. A 5 kg mass has coordinates of (3 m, 4 m), a 3 kg mass is at (2 m, 7 m). Where is the center of gravity? Draw a diagram, label and number the axes, and show calculated location of the x_{cg}, y_{cg} .

$$M_1 = 5 \text{ kg}$$

$$x_{1cm} = 3 \text{ m}$$

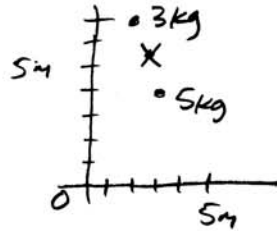
$$y_{1cm} = 4 \text{ m}$$

$$M_2 = 3 \text{ kg}$$

$$x_{2cm} = 2 \text{ m}$$

$$y_{2cm} = 7 \text{ m}$$

$$x_{cm}, y_{cm} = \text{---}, \text{---}$$



$$x_{cm} = \frac{M_1 x_1 + M_2 x_2}{M_1 + M_2}$$

$$= \frac{5 \text{ kg}(3 \text{ m}) + 3 \text{ kg}(2 \text{ m})}{5 \text{ kg} + 3 \text{ kg}}$$

$$x_{cm} = 2.63 \text{ m}$$

$$x_{cm}, y_{cm} = 2.63 \text{ m}, 5.13 \text{ m}$$

$$y_{cm} = \frac{5 \text{ kg}(4 \text{ m}) + 3 \text{ kg}(7 \text{ m})}{5 \text{ kg} + 3 \text{ kg}}$$

$$y_{cm} = 5.13 \text{ m}$$

3. A horizontal 40 N bar 2 m long is connected to a wall on the left end. At the far right end, an 80 N weight hangs from it. A cable is attached at the 1.5 m location and the other end of the cable is attached to the wall. The angle between the cable and the bar is 37° . (a) What is the tension in the cable? (b) What is the vertical force exerted on the bar by the wall and which direction is it?

$$F_b = 40 \text{ N}$$

$$l = 2 \text{ m}$$

$$F_m = 80 \text{ N}$$

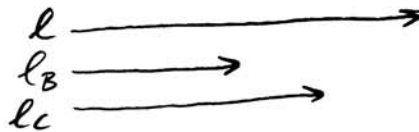
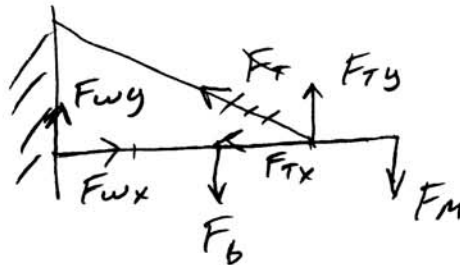
$$l_c = 1.5 \text{ m}$$

$$\theta = 37^\circ$$

$$F_T = \text{---} \text{ N}$$

$$F_{wy} = \text{---} \text{ N}$$

$$l_b = 1 \text{ m}$$



$$F_{wy} - F_b + F_T \sin \theta - F_m = 0$$

$$F_{wy} = F_b - F_T \sin \theta + F_m$$

$$= 40 \text{ N} - 222 \text{ N}(\sin 37^\circ) + 80 \text{ N}$$

$$\sum \vec{\tau} = 0 \quad F_{Ty} = F_T \sin \theta$$

$$-l_b F_b + l_c F_{Ty} - l F_m = 0$$

$$F_T = \frac{l_b F_b + l F_m}{l_c \sin \theta}$$

$$= \frac{1 \text{ m}(40 \text{ N}) + 2 \text{ m}(80 \text{ N})}{1.5 \text{ m}(\sin 37^\circ)}$$

a) $F_T = 222 \text{ N}$

b) $F_{wy} = -13.6 \text{ N}$ that is, down.

Honors Physics Test - Ch. 8 - Torque and Rotational Energy - 05-06-10b

Name WARD

$\tau = rF$ (if r or a component of r is perpendicular to τ) $K_t = \frac{1}{2}mv^2$ $K_r = \frac{1}{2}I\omega^2$ $U_g = mgh$ $\Sigma\tau = I\alpha$
 $I_{ss} = \frac{2}{5}mr^2$ $I_{hs} = \frac{2}{3}mr^2$ $I_{ring} = mr^2$ $I_{disk} = \frac{1}{2}mr^2$ $L = I\omega$ $v = r\omega$ $a = r\alpha$

List your givens and unknown(s) for each problem. Draw good force/torque diagrams where needed, using clearly labeled vectors

1. A 2 kg solid sphere with $r = 0.3$ m is mounted on an axle. A rope is wrapped around the outer edge of the sphere and pulled with a constant force of 5 N. (a) What torque is produced? (b) Find the angular acceleration of the sphere. (c) Find the linear acceleration of the rope.

$M = 2 \text{ kg}$ (solid sphere)

$r = 0.3 \text{ m}$

$F = 5 \text{ N}$

$\tau = \text{--- Nm}$

$\alpha = \text{--- } \frac{\text{rad}}{\text{s}^2}$

$a = \text{--- } \frac{\text{m}}{\text{s}^2}$

$\tau = rF$
 $= 0.3 \text{ m}(5 \text{ N})$

a) $\tau = 1.5 \text{ Nm}$

$\tau = I\alpha$

$\alpha = \frac{\tau}{I}$

$= \frac{1.5 \text{ Nm}}{\frac{2}{5}Mr^2}$

$= \frac{1.5 \text{ Nm}}{\frac{2}{5}(2 \text{ kg})(0.3 \text{ m})^2}$

b) $\alpha = 20.8 \frac{\text{rad}}{\text{s}^2}$

$a = r\alpha$

$= 0.3 \text{ m}(20.8 \frac{\text{rad}}{\text{s}^2})$

c) $a = 6.25 \frac{\text{m}}{\text{s}^2}$

- 2) An 8 kg hollow sphere with $r = 0.5$ m rolls down a 30° hill from a height of 5 meters. (a) Using energy conservation, derive the equation for the velocity of the sphere at the bottom of the hill. (b) Numerically calculate the linear velocity and the angular velocity of the sphere at the bottom of the hill. (EC) Do part b for a solid disk using the same mass and radius. You may re-use equation(s) from earlier parts.

2) $M = 8 \text{ kg}$ (hollow sphere)

$$r = 0.5 \text{ m}$$

$$\theta = 30^\circ$$

$$h = 5 \text{ m}$$

$$v_f = \frac{\text{m}}{\text{s}}$$

$$\omega_f = \frac{\text{rad}}{\text{s}}$$

$$U_{g_i} + K_{R_i} + K_{T_i} = U_{g_f} + K_{R_f} + K_{T_f}$$

$$mgh = \frac{1}{2} M v^2 + \frac{1}{2} \left(\frac{2}{3} M r^2 \right) \omega^2 v^2$$

$$gh = \frac{5}{6} v^2$$

$$v_f = \sqrt{\frac{6gh}{5}}$$

$$= \sqrt{\frac{6(9.8 \frac{\text{m}}{\text{s}^2})(5 \text{ m})}{5}}$$

$$b) \boxed{v_f = 7.67 \frac{\text{m}}{\text{s}}}$$

$$v = r\omega$$

$$\omega = \frac{v}{r}$$

$$= \frac{7.67 \frac{\text{m}}{\text{s}}}{0.5 \text{ m}}$$

$$b) \boxed{\omega_f = 15.3 \frac{\text{rad}}{\text{s}}}$$

Ec) Change $\frac{2}{3}$ to $\frac{1}{2}$

$$gh = \frac{1}{2} v^2 + \frac{1}{2} \left(\frac{1}{2} \right) v^2$$

$$gh = \frac{3}{4} v^2$$

$$v_f = \sqrt{\frac{4gh}{3}}$$

$$= \sqrt{\frac{4(9.8 \frac{\text{m}}{\text{s}^2})(5 \text{ m})}{3}}$$

$$b) \boxed{v_f = 8.08 \frac{\text{m}}{\text{s}}}$$

$$v = r\omega$$

$$\omega = \frac{v}{r}$$

$$= \frac{8.08 \frac{\text{m}}{\text{s}}}{0.5 \frac{\text{m}}{\text{s}}}$$

$$b) \boxed{\omega_f = 16.2 \frac{\text{rad}}{\text{s}}}$$

Honors Physics Test - Ch. 8 - Angular Momentum - 05-10-10c

Name WARD 0

$\tau = rF$ (if r or a component of r is perpendicular to τ) $K_t = \frac{1}{2}mv^2$ $K_r = \frac{1}{2}I\omega^2$ $U_g = mgh$ $\Sigma\tau = I\alpha$
 $I_{ss} = \frac{2}{5}mr^2$ $I_{hs} = \frac{2}{3}mr^2$ $I_{ring} = mr^2$ $I_{disk} = \frac{1}{2}mr^2$ $L = I\omega$ $\tau \Delta t = I \Delta\omega$ $I_i\omega_i = I_f\omega_f$ $v = r\omega$
 $a = r\alpha$

List your givens and unknown(s) for each problem. Draw good force/torque diagrams where needed, using clearly labeled vectors

3. A diver comes off the diving board with his arms straight up and his body in a straight line, while slightly falling forward. His rotational inertia is 11 kg m^2 and his angular speed is 0.75 rad/s . Later, he tucks his body into a ball and his new rotational inertia becomes 2.5 kg m^2 . (a) What is his new angular speed? (b) What is his final rotational kinetic energy? (c) Is his final rotational kinetic energy larger or smaller than when he first left the board? Explain. Use calculation or words. Or both. Also explain why his rotational kinetic energy changes.

$$\begin{aligned}
 \omega_i &= 0.75 \frac{\text{rad}}{\text{s}} & I_i \omega_i &= I_f \omega_f \\
 I_i &= 11 \text{ kg m}^2 & \omega_f &= \frac{I_i \omega_i}{I_f} \\
 I_f &= 2.5 \text{ kg m}^2 & &= \frac{11 \text{ kg m}^2 (0.75 \frac{\text{rad}}{\text{s}})}{2.5 \text{ kg m}^2} \\
 \omega_f &= \text{---} \frac{\text{rad}}{\text{s}} & & \\
 \text{a) } & \boxed{\omega_f = 3.3 \frac{\text{rad}}{\text{s}}}
 \end{aligned}$$

$$\begin{aligned}
 K_i &= \frac{1}{2} I_i \omega_i^2 & K_f &= \frac{1}{2} I_f \omega_f^2 \\
 &= \frac{1}{2} (11 \text{ kg m}^2) (0.75 \frac{\text{rad}}{\text{s}})^2 & &= \frac{1}{2} (2.5 \text{ kg m}^2) (3.3 \frac{\text{rad}}{\text{s}})^2 \\
 K_i &= 3.09 \text{ J} & \text{b) } & \boxed{K_f = 13.6 \text{ J}}
 \end{aligned}$$

- c) $\boxed{K_f > K_i}$ This is because the diver does work pulling his arms and legs in. This work shows up as an increase in kinetic energy.
 (It is not enough to say K increase because he is spinning faster. There is more involved than just that.)

- EC) A 2 kg sphere with radius of 0.15 m rotating clockwise (viewed from above) at 6 rad/s is dropped onto a 3 kg ring of radius 0.10 m rotating in the opposite direction at 3.5 rad/s. They end up spinning at the same ω . Calculate the new ω .

OVER

$$EC) M_1 = 2 \text{ kg (solid sphere)}$$

$$r_1 = 0.15 \text{ m}$$

$$\omega_1 = -6 \frac{\text{rad}}{\text{s}}$$

$$M_2 = 3 \text{ kg (ring)}$$

$$r_2 = 0.1 \text{ m}$$

$$\omega_2 = +3.5 \frac{\text{rad}}{\text{s}}$$

$$\omega_f = \text{---} \frac{\text{rad}}{\text{s}}$$

$$L_i = L_f$$

$$I_1 \omega_1 + I_2 \omega_2 = I_f \omega_f$$

$$\frac{2}{5} M_1 r_1^2 \omega_1 + M_2 r_2^2 \omega_2 = \left(\frac{2}{5} M_1 r_1^2 + M_2 r_2^2 \right) \omega_f$$

$$\omega_f = \frac{\frac{2}{5} M_1 r_1^2 \omega_1 + M_2 r_2^2 \omega_2}{\frac{2}{5} M_1 r_1^2 + M_2 r_2^2}$$

$$= \frac{\frac{2}{5} (2 \text{ kg}) (0.15 \text{ m})^2 (-6 \frac{\text{rad}}{\text{s}}) + 3 \text{ kg} (0.1 \text{ m})^2 (3.5 \frac{\text{rad}}{\text{s}})}{\frac{2}{5} (2 \text{ kg}) (0.15 \text{ m})^2 + 3 \text{ kg} (0.1 \text{ m})^2}$$

$$\boxed{\omega_f = -0.0625 \frac{\text{rad}}{\text{s}}} \text{ that is, clockwise}$$